Properties of Multiplication and Division and Solving Problems with 2-5 and 10

In this first module of Grade 3, we build on second grade knowledge of addition and work toward greater fluency. We will also be building arrays (arrangements of a set of objects organized into equal groups in rows and columns), and setting the stage for multiplication and division.

A number bond illustration of the Distributive Property:

\[ 9 \times 10 = (5 \times 10) + (4 \times 10) \]

Terms, Phrases, and Strategies in this Module:

- **Array**: a set of numbers or objects that follow a specific pattern, a matrix
- **Commutative Property**: e.g., rotate a rectangular array 90 degrees to demonstrate that factors in a multiplication sentence can switch places
- **Equal groups**: with reference to multiplication and division; one factor is the number of objects in a group, and the other is a multiplier that indicates the number of groups
- **Equation**: a statement that two expressions are equal, e.g., \( 3 \times 4 = 12 \)
- **Distributive Property**: e.g., \( 12 \times 3 = (10 \times 3) + (2 \times 3) \). The 3 is the multiplier and the 12 is decomposed into 10 and 2
- **Factors**: i.e., numbers that are multiplied to obtain a product
- **Quotient**: the answer when one number is divided by another

What Comes After this Module:

In Module 2, students will have opportunities to use tools that build both measurement skills as well as conceptual understanding of metric and time units. Through practical application of measurement skills, students will practice both estimating and rounding numbers.

An illustration of the Commutative Property

Key Common Core Standards:

- **Represent and solve problems involving multiplication and division**
  - Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities
- **Understand properties of multiplication and the relationship between multiplication and division**
  - Apply properties of operations as strategies to multiply and divide
  - Understand division as an unknown-factor problem
- **Multiply and divide within 100**
  - Fluently multiply and divide within 100
- **Solve problems involving the four operations, and identify and explain patterns in arithmetic**
  - Solve two-step word problems using the four operations

How you can help at home:

- Have your student set out groups of small objects in arrays (equal groups in rows and columns) and write the accompanying multiplication equation
- Encourage your student to practice multiplication facts for 2s, 3s, 4s, 5s, and 10s until they know them fluently

Prepared by Erin Schweng, Math Coach
Welcome to A Story of Units!

Each module’s parent tip sheet will highlight a new strategy or math model your student will be working on.

Read on to learn a little bit about Eureka Math, the creators of A Story of Units:

Eureka Math is a complete, PreK-12 curriculum and professional development platform. It follows the focus and coherence of the Common Core State Standards (CCSS) and carefully sequences the progression of mathematical ideas into expertly crafted instructional modules.

This curriculum is distinguished not only by its adherence to the CCSS; it is also based on a theory of teaching math that is proven to work. That theory posits that mathematical knowledge is conveyed most effectively when it is taught in a sequence that follows the “story” of mathematics itself. This is why we call the elementary portion of Eureka Math "A Story of Units." The sequencing has been joined with methods of instruction that have been proven to work, in this nation and abroad. These methods drive student understanding beyond process, to deep mastery of mathematical concepts.

The goal of Eureka Math is to produce students who are not merely literate, but fluent, in mathematics. Your student has an exciting year of discovering the story of mathematics ahead!

Sample Problem from Module 1:
(Example taken from Module 1, Lesson 7)

Anna picks 24 flowers. She makes equal bundles of flowers and gives 1 bundle to each of her 7 friends. She keeps a bundle for herself too. How many flowers does Anna put in each bundle?

Arrays: students worked with arrays toward the end of Grade 2, learning how to use them to show repeated addition. Now, in Grade 3, students put all of their knowledge to work as they learn multiplication and division skills, using arrays to demonstrate the properties of both operations.

(Below) A simple teddy bear array for 3 x 4, or three rows with four in each row

(Left) An array with multiple rows of 3 in each row, showing foundation for multiplication as repeated addition

For more information visit commoncore.org
Place Value and Problem Solving with Units of Measure

This module will tie our place value learning to some real-world work with measurement using the metric system. Students will also work on telling time and solving problems relating to elapsed time.

What Came Before this Module: We deeply explored the meaning of multiplication and division, working from concrete to abstract examples.

What Comes After this Module: We will continue our work on multiplication and division, this time working to build our knowledge of units of 6, 7, 8, and 9, as well as multiples of 10.

Thinking mathematically is hard but important work!

Key Words to Know

Important Metric Words:
- Gram (g)
- Kilogram (kg)
- Liter (L)
- Milliliter (mL)
- Centimeter (cm)
- Meter (m)

Other math terms:
- Analog clock: a clock that is not digital
- Capacity: the amount that a container can hold
- Compose: change 10 smaller units for 1 of the next unit on the place value chart
- Interval: time passed, or a segment on the number line
- Plot: locate and label a point on the number line
- Point: a specific location on the number line
- Round: estimate a number to the nearest 10 or 100 using place value

Key Common Core Standards:

- Use place value understanding and properties of operations to perform multi-digit arithmetic
  - Round numbers to the nearest 10 or 100
  - Fluently add and subtract within 1000

- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects
  - Tell and write time to the nearest minute and measure time intervals
  - Measure and estimate liquid volume and mass of objects

How you can help at home:

- Ask your student to help with all kinds of measurement around the house
- Continue to practice telling time, and begin to ask questions about elapsed time, e.g., “How many minutes have passed since we got home from school?”
A Story of Units has several key mathematical “models” that will be used throughout a student’s elementary years.

The number line is a powerful, flexible model that students can use in many ways. In this particular module, students make frequent use of both vertical and horizontal number lines, learning to find endpoints and mark exactly halfway in between them, finding elapsed time, and using them on measuring containers.

As students move through the grades, number lines can be used to examine the relationships between numbers in ever more detailed ways, including decimals, fractions, and eventually positive and negative numbers. See how many number lines you and your student can spot around you at home!

Sample Problem from Module 2:
(Example taken from Lesson 13, Module 2)

Here is a sample elapsed time problem that can be solved with a number line:

The school ballet recital begins at 12:17 p.m. and ends at 12:45 p.m. How many minutes long is the ballet recital?

20 + 8 = 28 minutes.

The ballet recital took 28 minutes.
### Key Common Core Standards:

- Represent and solve problems involving multiplication and division
- Understand properties of multiplication and the relationship between multiplication and division
- Multiply and divide within 100
- Solve problems involving the four operations
- Use place value understanding and properties of operations to perform multi-digit arithmetic

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**Multiplication and Division with Units of 0, 1, 6–9, and Multiples of 10**

In this module we will go deep into our learning about these two related operations. Students will practice their math facts to become fluent, and will learn several strategies for multiplying and dividing numbers.

**Key Words to Know**

- **Array**: a set of numbers or objects that follow a specific pattern
- **Commutative Property**: e.g. $3 \times 2 = 2 \times 3$
- **Distributive Property**: e.g. $12 \times 3 = (10 + 2) \times 3 = (10 \times 3) + (2 \times 3)$
- **Factors**: numbers that are multiplied to obtain a product
- **Multiple**: e.g. multiples of 9 are 18, 27, 36, 45, etc.
- **Number bond**: model used to show part-part-whole relationships
- **Product**: the quantity resulting from multiplying factors
- **Quotient**: the answer when one number is divided by another
- **Tape diagram**: a method for modeling problems

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**What Came Before this Module**: We learned more about both measurement and the place value system. We also worked with telling time to the nearest minute and elapsed time.

**What Comes After this Module**: We will extend our multiplication skills by studying area and two-dimensional spaces. We will design a floor plan and calculate the area using our multiplication skills.

**How you can help at home**:

- Continue to review multiplication and division math facts with your student.
- Help your student notice related math facts, e.g. $4 \times 2 = 8$, $4 \times 20 = 80$, $40 \times 2 = 80$.
A Story of Units has several key mathematical “models” that will be used throughout a student’s elementary years.

The tape diagram is a powerful model that students can use to solve various kinds of problems. In earlier grades, tape diagrams are models of addition and subtraction, but now in third grade we will use them to model multiplication and division as well. Tape diagrams are also called “bar models” and consist of a simple bar drawing that students make and adjust to fit a word problem. They then use the drawing to discuss and solve the problem.

As students move through the grades, tape diagrams provide an essential bridge to algebra. Below is a sample word problem from Module 3 solved using a tape diagram to show the parts of the problem.

Module 3 Sample Problem

Asmir buys 8 boxes of 9 candles for his dad’s birthday. After putting some candles on the cake, there are 28 candles left. How many candles does Asmir use?

(Example taken from Lesson 11)
Multiplication and Area

In this 20-day module, students explore area as an attribute of two-dimensional figures and relate it to their prior work with multiplication. Students will build understanding that a 2x6, 1x12, and 3x4 rectangle each have the same area, and will learn how to calculate the area of a floor plan of their own design.

Key Terms and Ideas

New Terms:
Area - the amount of two-dimensional space inside a bounded region
Area model - a model for multiplication that relates rectangular arrays to area
Square unit - a unit of area (could be square centimeters, inches, feet, or meters)
Tile (as a verb) - to cover a region without gaps or overlaps
Unit Square - whatever the length unit (e.g. centimeters, inches), a unit square is a 1 unit by 1 unit square of that length
Whole Number - an integer number without fractions

Terms to Review:
Array
Commutative Property
Distribute
Length
Multiplication

What Came Before this Module: We worked extensively on relating multiplication and division, learned several different strategies for those operations, and practiced our math facts.

What Comes After this Module: We will begin to formalize our understanding of fractions as equal parts of a whole, using the number line as well as area models to support our learning.

Key Common Core Standards:

- Geometric Measurement: understand concepts of area and relate area to multiplication and to addition
  - A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area
  - Measure areas by counting unit squares
  - Relate area to the operations of multiplication and addition

How you can help at home:

⇒ Continue to review multiplication and division math facts with your student
⇒ Practice drawing simple two-dimensional rectangular shapes and calculating the area using multiplication
A Story of Units has several key mathematical “models” that will be used throughout a student’s elementary years.

Students began in earlier grades to build arrays, showing multiplication and division as a series of rows and columns. In 3rd grade, they begin the transition to understanding these types of problems in the context of an area model.

As students move through the grades, the area model will be a powerful tool that can take them all the way into algebra and beyond. One of the goals in A Story of Units is to first give students concrete experiences with mathematical concepts, and then build slowly toward more abstract representations of those concepts. The area model is a tool that helps students to make that important leap.

Module 4 Sample Problem
(Example taken from Lesson 13)

Anil finds the area of a 5-inch by 17-inch rectangle by breaking it into 2 smaller rectangles. Show one way that he could have solved the problem.

What is the area of the rectangle?

Possible Solution:

\[
\begin{align*}
5 \times 17 &= (5 \times 10) + (5 \times 7) \\
5 \times 17 &= 50 + 35 \\
5 \times 17 &= 85
\end{align*}
\]

The area of the rectangle is 85 sq. in.
**Eureka Math tips for Parents**

**Fractions as Numbers on the Number Line**

In this 35-day module, students extend and deepen 2nd grade practice with “equal shares” to understanding fractions as equal partitions of a whole. They formalize their knowledge as they work with area models and the number line.

Students will learn to partition number lines into fractional parts, renaming whole numbers as fractions.

```
0 1/2 1 3/2 2
```

**What Came Before this Module:** Students explored area as an attribute of two-dimensional figures and related it to their prior work with multiplication.

**What Comes After this Module:** In Module 6, students will begin work on data collection and representation. Specifically, students will generate and analyze categorical and measurement data.

```
0 1/2 1 3/2 2
```

**Key Terms and Ideas**

**New Terms:**
- Unit fraction: fractions with numerator of 1
- Non-unit fraction: fractions with numerators other than 1
- Fractional unit: half, third, fourth, etc.
- Equal parts: parts with equal measurements
- Unit interval: the interval from 0 to 1, measured by length
- Equivalent fraction: fractions that are the same size, or the same point on a number line
- Copies: refers to the number of unit fractions in one whole

**Terms and Symbols to Review:**
- Number Line
- Arrays
- Equal Shares
- Whole
- Fraction
- Partition
- =, <, >

**Key Common Core Standards:**

- **Develop understanding of fractions as numbers**
  - Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts; understand a fraction $a/b$ as the quantity formed by a parts of size $1/b$
  - Understand a fraction as a number on the number line; represent fractions on a number line diagram.
  - Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

- **Reason with shapes and their attributes**
  - Partition shapes into parts with equal areas

Prepared by Erin Schweng, Math Coach
**A Story of Units** has several key mathematical “models” that will be used throughout a student’s elementary years.

The number bond is a pictorial representation of part/part/whole relationships showing that smaller numbers (the parts) make up larger numbers (the whole). The number bond is a key model for showing students how to both take apart (decompose) and put together (compose) numbers.

Students become familiar with number bonds in Kindergarten, and they are used repeatedly throughout the grades in various situations. In Grade 3, students compose fractional numbers using number bonds as a powerful tool to see the unit fractions that make up a whole number. They will also use number bonds to decompose whole numbers greater than 1 into fractional parts.

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**Module 5 Sample Problem**
(Example taken from Lesson 22)

Mr. Ramos wants to nail the TV cord against the wall so no one trips. He puts 7 nails equally spaced along the cord. Draw a number line representing the cord. Label it from 0 at the start of the cord to 1 at the end. Put a mark where Mr. Ramos puts each nail with a fraction.

a. Build a number bond with unit fractions to 1 whole.

b. Write the fraction of the nail that is equivalent to $\frac{1}{2}$ the cord.